Some Pitfalls in Millimeter-Wave Noise Measurements Utilizing a Cross-Correlation Receiver

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***Abstract*—It is shown that the use of a hybrid junction as the power splitter in a cross-correlation receiver for millimeter-wave (94-GHz) noise measurements at low temperature (2 K) introduces an unwanted source of noise which defeats the purpose of the measurements. The same is the case when isolators are introduced to prevent cross-coupling of the noise from the two receiver channels.**

I. Introduction

WE ARE IN THE process of acquiring the instrumentation for conducting noise measurements at 94 GHz upon solid-state devices immersed in a cryostat at temperatures on the order of 2 K, in order to detect quantum noise effects. In such cases, the noise due to the device under test (DUT) is "buried" in the noise caused by instrumentation (at room temperature) utilized to measure it. One method contemplated for overcoming this problem was the use of the cross-correlation measurement approach depicted in Fig. 1. In principle, it works as follows.

Noise from the DUT is split equally into two fully correlated components which are processed by two ideally identical 94-GHz single-sideband receivers whose outputs provide 1–100-MHz "windows" containing the fully correlated noise to be measured "buried" in fully uncorrelated noise introduced by each receiver. The four-quadrant multiplier and integrator serve to measure the cross correlation of the two output noise signals. The noise from the two receiver channels, being uncorrelated, results in a net zero output when the integration is performed over a sufficiently long time interval. The desired noise from the DUT, being fully correlated, produces a nonzero output, and thus becomes measurable.

The local oscillators (LO's) of (at least) the first mixers of each receiver cannot be common to both because that would introduce unwanted correlated noise in each channel. However, those LO's must be phase-locked in order to preserve the phase information of the noise due to the DUT. The isolators shown in Fig. 1 are necessary to

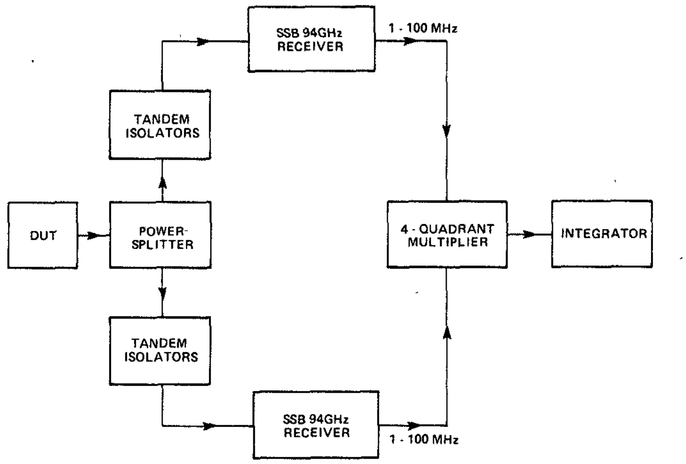

Fig. 1. Schematic of a proposed 94-GHz correlation receiver for low temperature quantum noise measurements.

prevent receiver noise from channel "A" entering channel "B", and vice versa, because that would also introduce unwanted correlated noise in the two channels.

The realization of such a correlation receiver at 94 GHz is nontrivial and certainly expensive. In the process of defining how this was to be done, we have discovered that one can *not* utilize a hybrid junction as the power splitter, nor can one utilize the isolators required for the measurement to succeed. It is the purpose of this paper to expose why this is the case.

II. Analysis

To avoid second order complication of the signal flow graphs, we assume ideal components, i.e., a perfect hybrid, perfect circulators, perfect resistive terminations, a perfect sliding short, etc. These assumptions are clearly unrealistic, but they do not mask the fundamental nature of the problems to be addressed.

Fig. 2 shows such a hybrid, in the form of a magic tee, enumerates its four ports, and gives the signal flow graph which results from its scattering matrix equation which is [1]

$$\begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \end{bmatrix} = \begin{bmatrix} 0 & \tau & \tau & 0 \\ \tau & 0 & 0 & \tau \\ \tau & 0 & 0 & -\tau \\ 0 & \tau & -\tau & 0 \end{bmatrix} \cdot \begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \end{bmatrix} \quad (1)$$

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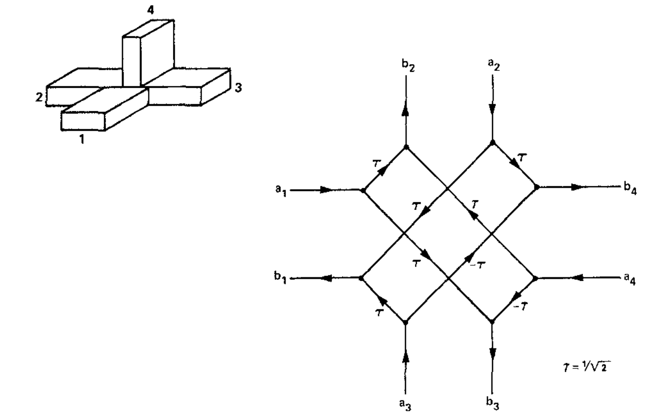

Fig. 2. A 4-port hybrid and its idealized signal flow graph

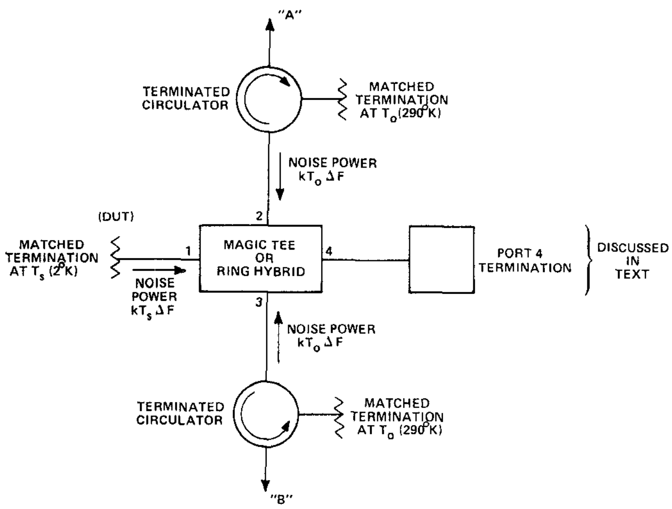

Fig. 3. Implementation of the front end of a correlation receiver using a hybrid and isolators.

where $\tau = 1/\sqrt{2}$. Fig. 3 shows the use of that hybrid as the power splitter of our correlation receiver, where we show the DUT connected at its port 1 as a simple matched resistive termination at temperature T_s (which we intend to set $\simeq 2$ K). Two terminated 3-port circulators (the first of a chain of several), serving as isolators, are shown connected to ports 2 and 3 of the hybrid. The termination connected to port 4 will be the subject of discussion. Note that the two terminated circulators, at room temperature T_0, cause full Nyquist noise power $kT_0\Delta f$ to be incident upon ports 2 and 3.

A. Port 4 Resistively Terminated (Matched) at Temperature T_s

Assume that port 4 of the hybrid junction is matched with a resistive termination at temperature T_s (i.e., the hybrid, the resistive match at port 1 (the DUT), and the resistive match at port 4 are all immersed in the cryostat). In this case, the Nyquist noise incident upon ports 2 and 3 causes no problems. The signal flow graph of Fig. 2 shows such noise to split equally between ports 1 and 4 where it is absorbed because those ports are matched.

Let b_{N1} be the noise phasor incident upon port 1 from the DUT (i.e., $\langle b_{N1}b_{N1}^*\rangle = kT_s\Delta f$) and b_{N4} be that incident upon port 4 from its resistive termination (i.e., $\langle b_{N4}b_{N4}^*\rangle = kT_s\Delta f$ also).[1] These two noise phasors are obviously uncorrelated. The signal flow graph of Fig. 2 shows that the instantaneous noise phasor emanating from "A" in Fig. 3 is then $\tau(b_{N1}+b_{N4})$, while that emanating from "B" is $\tau(b_{N1}-b_{N4})$. After processing via ideally identical receiver channels, then being multiplied and time-averaged by the cross correlator, the output signal S of the latter will be (ignoring the gain of the receivers)

$$\begin{aligned} S &= \tau^2\langle (b_{N1}+b_{N4})(b_{N1}^* - b_{N4}^*)\rangle \\ &= \tau^2[\langle b_{N1}b_{N1}^*\rangle - \langle b_{N4}b_{N4}^*\rangle] \\ &= \tau^2[kT_s\Delta f - kT_s\Delta f] \equiv 0. \end{aligned} \tag{2}$$

This result occurs because the cross-correlation terms $\langle b_{N1}b_{N4}^*\rangle$ and $\langle b_{N1}^*b_{N4}\rangle$ vanish since b_{N1} and b_{N4} are fully uncorrelated. Since our objective is to extract from this measurement an output which is a measure of the noise $\langle b_{N1}b_{N1}^*\rangle$ caused by the DUT alone, we see that this approach fails miserably to do so!

We conclude that terminating port 4 of the hybrid junction with a matched resistive load will not work. Let us, therefore, examine the obvious alternative of terminating that port reactively.

B. Port 4 Reactively Terminated by a Perfect Sliding Short

For generality, assume that port 4 is now terminated with a tunable lossless short so that the reflection coefficient Γ_4 seen looking out of port 4 is $\exp(j\theta_4)$, with θ_4 taking on all possible values between 0 and 2π. The noise sources from the two terminated circulators at ports 2 and 3 can now no longer be ignored since we have already seen that their signals are diverted, in part, to port 4 where they will now be reflected, not absorbed, as was the case above. Accordingly, let b_{N2} be the noise phasor due to the isolator connected to port 2 (i.e., $\langle b_{N2}b_{N2}^*\rangle = kT_0\Delta f$) and b_{N3} be that due to the other isolator (i.e., $\langle b_{N3}b_{N3}^*\rangle = kT_0\Delta f$ also). These noise phasors are also obviously uncorrelated.

It is helpful, for the purpose of analyzing this new situation, to introduce a revised signal flow graph which explicitly displays the three sources of noise which now came into play. This is shown in Fig. 4, where we assume that ports 1, 2, and 3 are still matched as before. From that flow graph, we now find for the noise phasors emanating from ports 1, 2, and 3

$$b_1 = \tau[b_{N2} + b_{N3}] \tag{3}$$

$$b_2 = \tau[b_{N1} + \tau\Gamma_4 b_{N2} - \tau\Gamma_4 b_{N3}] \tag{4}$$

$$b_3 = \tau[b_{N1} - \tau\Gamma_4 b_{N2} + \tau\Gamma_4 b_{N3}]. \tag{5}$$

The noise power $\langle b_1 b_1^*\rangle = \tau^2[\langle b_{N2}b_{N2}\rangle + \langle b_{N3}b_{N3}^*\rangle]$ exiting port 1 is absorbed by its resistive termination (the DUT). Since $\tau = 1/\sqrt{2}$, we see that half of the Nyquist

[1] In this discussion, we ignore the quantum correction factor for the Nyquist noise. The reader can, if he wishes, picture T_s as being an "effective" temperature which includes that correction factor.

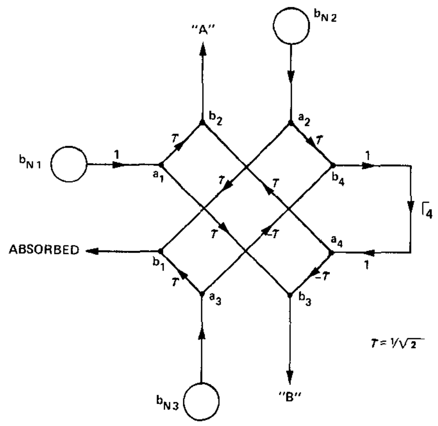

Fig. 4. Signal flow graph for the noise performance of the receiver front end of Fig. 3 when port 4 of the hybrid is terminated with an ideal tunable short.

noise $kT_0\Delta f$ due to each terminated circulator is so absorbed. However, the remaining half is split equally between ports 2 and 3. For example, the noise power exiting from port 2, obtained from (4), is now

$$\langle b_2 b_2^* \rangle = \tau^2 [\langle b_{N1} b_{N1}^* \rangle + \tau^2 \langle b_{N2} b_{N2}^* \rangle + \tau^2 \langle b_{N3} b_{N3}^* \rangle] \quad (6)$$

where we have used the fact that $\Gamma_4 \Gamma_4^* = 1$, and have recognized that all three noise sources are uncorrelated. An identical expression is obtained for the noise power $\langle b_3 b_3^* \rangle$ exiting port 3.

From the above, we conclude that the effect of terminating port 4 reactively, regardless of the phase of the reflection coefficient Γ_4, is that one-fourth of the Nyquist noise power from each of the terminated circulators at ports 2 and 3 is reflected, the other one-fourth being transmitted to their counterparts. Thus, we no longer have the perfect isolation between ports 2 and 3 which occurred when port 4 was resistively matched. We can expect correlated noise of equal magnitude at "A" and "B" in Fig. 3 from both isolators.

The resulting output of the cross correlator under these conditions might well be anticipated. Equation (4) gives the noise phasor emanating from "A", while (5) gives that emanating from "B". Again ignoring receiver gain, the output of the cross correlator will be

$$S = \langle b_2 b_3^* \rangle = \tau^2 [\langle b_{N1} b_{N1}^* \rangle - \tau^2 \langle b_{N2} b_{N2}^* \rangle - \tau^2 \langle b_{N3} b_{N3}^* \rangle] \quad (7)$$

where we have again used the fact that $\Gamma_4 \Gamma_4^* = 1$, and that the three noise phasors b_{N1}, b_{N2}, and b_{N3} are all uncorrelated. Since $\tau = 1/\sqrt{2}$, $\langle b_{N1} b_{N1}^* \rangle = kT_s \Delta f$, while $\langle b_{N2} b_{N2}^* \rangle = \langle b_{N3} b_{N3}^* \rangle = kT_0 \Delta f$, (7) translates into

$$S = k(T_s - T_0)\Delta f/2. \quad (8)$$

With $T_s = 2$ K, $T_0 = 290$ K, we conclude that our dual-channel receiver and cross correlator will produce a negative dc output which very successfully measures the noise of a resistor at *room temperature*! Note that even if it were possible to immerse the two terminated circulators in the cryostat at temperature T_s, the problem would not go away, for (8) shows that the cross correlator would then produce *zero* output, as was the case when we matched port 4 resistively at temperature T_s in subsection A above.

III. Discussion

The principal advantage of using a hybrid junction as a power splitter is that, in principle, it provides perfect isolation between ports 2 and 3 when port 4 is matched. (In practice, 30 dB of isolation, minimum, is achievable at 94 GHz per manufacturer's specifications.) But when the "signal" power to be so split is *noise* itself, that advantage disappears. The analysis which we have presented shows that the Nyquist noise introduced by the matched resistive termination *necessary* at port 4 to achieve such isolation between ports 2 and 3 completely defeats the ability to measure the noise of the DUT entering port 1 *unless* the latter is at a level well in excess of the Nyquist noise of the resistive termination necessary at port 4. Since *one* of the objectives of our proposed quantum noise experiments is to measure the quantum correction factor of the Nyquist noise of a resistor itself at 94 GHz, 2 K (among other, more interesting solid-state devices), the difficulties encountered with a matched hybrid which we have uncovered are clearly unacceptable for our purposes.

We have shown that when port 4 of the hybrid is terminated reactively (regardless of the phase of its reflection coefficient Γ_4), the (idealized) isolation between ports 2 and 3 is reduced to 6 dB. One-half of the power incident upon port 2, for example, is absorbed at port 1 (assuming that it is perfectly matched). One-fourth of that incident power is reflected, the remaining one-fourth being transmitted to its counterpart, port 3. Similar results are obtained for the power incident upon port 3. Insofar as our noise measurements are concerned, this means that there is some inevitable correlated noise appearing in both receiver channels caused by whatever noise sources "downstream" of ports 2 and 3 cause noise power to be incident on those ports.

With regard to the comment just made, we have demonstrated that the utilization of terminated 3-port circulators "downstream" of ports 2 and 3, serving as isolators, produces intolerable results. In that case, the full Nyquist noise of the termination of the circulator is incident upon ports 2 and 3 of the hybrid. Even if said circulators were to be immersed in the cryostat at 2 K, the effect of their noise would still mask the noise of the resistive DUT which we have used in the above exposition.

Our analysis has utilized terminated circulators for the isolators because it is obvious that they are sources of Nyquist noise incident upon ports 2 and 3 of the hybrid. Other implementations of an isolator are possible, e.g., the two-port Faraday rotation type described in [2]. However, Siegman [3], using a thermodynamic argument, has shown that *any* ideal isolator is equivalent to an ideal 3-port circulator with its third port terminated insofar as its inherent noise is concerned. It is shown in a companion paper [4] that this is indeed the case for one such two-port isolator commonly encountered, i.e., that utilizing Faraday rotation in a ferrite-loaded section of circular waveguide joined at each end by sections of rectangular waveguide supporting only the TE_{10} mode, one of which incorporates a 45° twist.

In deriving (2), which shows perfect cancellation of the time-averaged Nyquist noise from the resistive terminations at ports 1 and 4 of the hybrid junction, we assumed identical temperatures T_s for both. Having shown that the resistive termination of port 4 is *essential* for the proper operation of the hybrid, one is led naturally to inquire whether something can be retrieved by operating the termination at port 4 at a temperature T_{s4} differing from that at port 1, T_{s1}, e.g., 4 K vis-a-vis 2 K. At those cryogenic temperatures, and with the tests being conducted at 94 GHz in order to display the quantum effects which we strive to measure, the Nyquist noise becomes

$$kT_s\Delta f \to hf\Delta f\left\{\left[\exp\left(\frac{hf}{kT_s}\right)-1\right]^{-1}+\frac{1}{2}\right\}. \qquad (9)$$

Certainly, the differing temperatures of the two resistors will cause the leading term within the brackets of (9) to differ, and the cancellation of the contributions from the two terminations exposed in (2) will not occur. *But* the factor 1/2 within those brackets, representing the zero-point energy contribution to the quantum correction, will still cancel in (2). Since one of the objectives of our research program is to determine the presence or absence of the zero-point energy term in the quantum correction factor for Nyquist noise, it is clear that the use of the elaborate cross-correlation approach which we originally envisioned would have been doomed to fail. An alternative approach, utilizing a single-channel millimeter-wave receiver, described in [5], is now being implemented.

IV. Conclusion

Dual-channel receivers implementing the cross-correlation technique have been successfully utilized in both radiometers and radio telescopes [6], [7]. But in those cases, two separate antennas aimed at the "target" provide the necessary isolation between the two receiver channels, and no hybrid power splitter is involved, unlike the situation which we have described. In the latter case, we have shown unequivocally that the cross-correlation approach cannot produce the results desired.

References

[1] R. E. Collin, *Foundations for Microwave Engineering.* New York: McGraw-Hill, 1966, ch. 6, pp. 282–285.
[2] R. E. Collin, *Foundations for Microwave Engineering.* New York: McGraw-Hill, 1966, ch. 6, pp. 301–302.
[3] A. E. Siegman, "Thermal noise in microwave systems, part I," *Microwave J.*, vol. 4, pp. 81–90, Mar. 1961.
[4] A. D. Sutherland, "On the inherent noise of an ideal two-port isolator," *IEEE Trans. Microwave Theory Tech.*, this issue, pp. 830–832.
[5] A. van der Ziel and A. D. Sutherland, "Measuring the quantum correction at zero bias in metal-oxide-metal diode noise," *IEEE J. Quantum Electron.*, vol. 18, pp. 17–19, Jan. 1982.
[6] S. J. Goldstein, Jr., "A comparison of two radiometer circuits," in *Proc. IRE*, vol. 43, pp. 1663–1666, Nov. 1955.
[7] M. E. Tiuri, "Radio astronomy receivers," *IEEE Trans. Antennas Prop.*, vol. AP-12, pp. 930–938, Dec. 1964.

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